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Fast window fusion using fuzzy equivalence relation

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ABSTRACT

Current window fusion of the sliding window based human detection is rather slow. This paper proposes a fast fuzzy equivalence relation based method (FER). It merges candidate windows based on the fuzzy equivalence relation structured from the normal fuzzy similarity relation. Experimental results demonstrate that the method can merge candidate windows faster than the popular non-maximum suppression based method (NMS) and the bounding region method (BR), while maintaining the detection quality.

1. Introduction

Sliding window strategy has been widely adopted as the main method to detect candidate humans in pedestrian detection research (Enzweiler and Gavrila, 2009; Gerónimo et al., 2010). It slides a detection window to search the human with a human detector. Normally there will be many candidate windows obtained, therefore, a post-processing of fusing all candidate windows into the final detections is required. This paper studies this window fusion problem and proposes a new and fast fuzzy equivalence relation based method (FER).

There have been some studies on window fusion, such as the heuristic fusion method (Rowley et al., 1996), the bounding region method (BR) (Viola and Jones, 2001,Viola and Jones, 2004), the response based method (Schneiderman and Kanade, 2004) and the non-maximum suppression method (NMS) (Dalal, 2006). As far as we know, there still lacks of a proper method to measure their performances partially due to their dependences on the previous detection steps. But at least one feature of those method we can measure, fusion speed. In fact, some of them are faster than others, e.g., BR is faster than NMS. Speed may affect the whole detection performance especially in real-time scenario. Therefore it is worthwhile to propose a fast window fusion method without sacrificing the performance. To this end, we adopt the fuzzy set theory and propose the FER method based on the fuzzy equivalence relation. Our experiments show that this method can merge windows efficiently in a faster speed than NMS and BR.

In the following, the related studies are introduced in Section 2 with the related fuzzy set theories briefly reviewed in Section 3. Section 4 discusses the FER method in detail. The experimental results are presented in Section 5 and the whole paper is concluded in Section 6.

2. Related work

In this section, we first review fuzzy set and fuzzy clustering studies and then introduce the literatures on window fusion for sliding window based human detection.

2.1. Fuzzy set

Fuzzy set theory was proposed by Zadeh, 1965 as an extension of the classical notion of set. In the classical set theory, an element either belongs or does not belong to a set. In contrast to such a two-valued logic, fuzzy set theory permits an element to partially belong to a set, where the partiality is valued between 0 and 1. Since then, fuzzy set theory is studied intensively, e.g., (de Glas, 1983; Novák et al., 1999; Piegat, 2005; Rezaei et al., 2006; Mendel, 2007; Ruspini, 2012). Recently Zadeh, 2008 discussed the importance of fuzzy logic from the nontraditional perspective and explained its important features: graduation, granulation, precisiation and the concept of a generalized constraint. He concluded that fuzzy logic has a high precisiation power.

Fuzzy set theory has been widely applied to different domains for incomplete or imprecise information (Zimmerman, 2010), e.g., control, clustering, data mining, decision, optimization. We are interested in fuzzy clustering (Höppner et al., 1999; de Oliveira and Pedrycz, 2007) which classifies objects according to their membership levels. Especially the hierarchical clustering methods

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based on fuzzy equivalence relation (Klir et al., 1995; Delgado et al., 1996) are attractive for the window fusion purpose because they do not need specify the cluster numbers.

2.2. Fuzzy clustering

Perhaps the most popular fuzzy clustering method is fuzzy c-means (FCM) (Dunn, 1973; Bezdek, 1981). FCM clusters data by optimizing an objective function measuring the similarities between the data and their centers. There are some improvements of FCM in the literatures, e.g., the fuzzy possibilistic c-means model and algorithm (Pal et al., 2005), the generalized FCM method (Jian and Yang, 2005), the kernel-based FCM methods (Graves and Pedrycz, 2010) and the multiple kernel FCM method (Huang et al., 2012). The FCMs-based methods need pre-specified desired cluster numbers and, therefore, are inconvenient whenever the desired number can not be determined in advance.

The shortcoming of FCMs-based methods can be overcomed by the hierarchical clustering using fuzzy equivalence relation (Klir et al., 1995; Delgado et al., 1996). However, the original clustering methods requires evaluating an accurate fuzzy equivalence relation which is difficult to derive directly. Lee, 1999 proposed to use the transitive closure as the fuzzy equivalence relation, which is computed from the normal fuzzy similarity relation. This idea is popular although there are alternative methods (Mirzaei and Rahmati, 2010). Mirzaei and Rahmati, 2010 further presented an iterative procedure to combine hierarchically clustered results without mismatch based on combining dendrogram-description matrices. Their idea is also applied by Ciaramella et al., 2011 to simulate the atmospherical phenomena.

Some other researches can be utilized for efficient fuzzy clustering. For example, Lee, 2001 and De Meyer et al., 2004 proposed new algorithms for computing the transitive closure; Le Capitaine, 2012 and Rezaei et al., 2006 proposed new similarity measures.

Fuzzy clustering can be applied to various areas, including text mining (Deng et al., 2010), astronomical data mining (Sessa et al., 2002), document clustering (Miyamoto, 1539), image segmentation (Naz et al., 2010), image retrieval (Ooi and Lim, 2009), etc. More in-depth discussion on the recent development can be found in (de Oliveira and Pedrycz, 2007).

However, as far as we know, there is no study of applying fuzzy clustering to the window fusion. Window fusion is to merge candidate windows into final correct detection windows by clustering similar ones. Apparently, it also belongs to the clustering problem and can be solved by fuzzy clustering. Therefore, we study the application of fuzzy clustering to the window fusion. We believe some relatively simple techniques are enough for our purpose because the candidate windows for putative pedestrians are normally sparse. Specially, in our method, (1) the idea of hierarchical clustering using fuzzy equivalence relation for unspecified number of clusters is adopted and simplified to compute only one transitive closure, therefore, no clustering combination in previous researches is needed; and (2) the traditional matrix method (De Meyer et al., 2004; Mirzaei and Rahmati, 2010) is used to obtain the key component – the transitive closure, considering the relatively small size of the fuzzy similarity matrix.

2.3. Window fusion

Most existing studies on window fusion compare the properties of the candidate windows directly. Rowley et al., 1996 decided the real face window simply based on the number of candidate windows in the neighboring area. A face window is confirmed only when the number is bigger than a pre-defined threshold. Viola and Jones, 2001,Viola and Jones, 2004 partitioned the candidate windows into disjoint subsets and merged the windows into the same subset if their bounding regions overlap. Therefore we call it as bounding region method (BR). The final true windows of BR are computed by averaging all borders of the overlapping windows. Schneiderman and Kanade, 2004 obtained the true window by searching the highest response in the circular neighboring area.

These direct methods are generally simple, intuitive and easy to implement. For them, only one final window is obtained within the neighboring area when candidates are very close or partially overlapped. Therefore it is prone to misclassification. Different scales of candidates are not considered for better discrimination.

Recently Dalal, 2006 proposed a new method called non-maximum suppression (NMS). In this method, window fusion is taken to be a kernel density estimation problem and treated as a suppression of non-maximum responses. Each detection is depicted by a 3-D position and scale space, and the mean-shift mode seeking method is used to localize the final detection. This method can effectively detect targets appearing in different scales and thus reduce classification errors. Therefore non-maximum suppression has been widely used in human detection related research (Wang and Lien, 2007; Bourdev et al., 2010; Parikh and Lawrence Zitnick, 2011). But the major drawback is the high computation complexity due to the mean-shift based clustering.

To speed up the NMS without degrading the fusion performance, we propose the FER method, which is based on fuzzy equivalence relation. Our experiments show that FER is significantly faster than NMS and BR.

3. Review of related fuzzy set theories

In this section, we review some related basic fuzzy set theories: fuzzy set, fuzzy relation, fuzzy equivalence relation and α-cut. For more details on fuzzy set theory and its application, please refer to Zadeh, 1965, Klir et al., 1995 and Xu et al., 2007.

3.1. Fuzzy set

A set is a collection of objects and an object in the set is called an element. In the classical set theory, the element either belongs to (*true*) or does not belong to (*false*) the set.

However, a logic based on the two values, '*true*' and '*false*', is sometimes inadequate when describing human reasoning. Therefore Zadeh proposed the fuzzy set as an extension of the classical set. Every element x of a fuzzy set A has a varying degree of membership $\Phi_A(0 \leqslant \Phi_A(x) \leqslant 1)$ where the value 1 or 0 means x is fully included in A or not included in A.

The element of a classical set is either in the set with membership degree '1' or out of the set with membership degree '0'. Therefore, the classical set is a special case of the fuzzy set. The classical set operations can be extended to the fuzzy set and therefore a series of fuzzy set operations are available.

3.2. Fuzzy relation

In the classical set, the relationship between elements (classical relation) is only in two degrees: '*completed related*' (1) or '*not related*' (0). It is built on the Cartesian product which is defined as a n-tuple set for n sets $A_i(1 \leqslant i \leqslant n)$

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) | a_i \in A_i,\ i = 1, \ldots, n\}.$$

The classical relation is a subset of the Cartesian product, which has the basic set operations, including union, intersection, etc.

Fuzzy relation, on the other hand, takes on varying degrees of relationship between 1 and 0. Let $a_i(1 \leqslant i \leqslant n)$ represent the elements from A_i. Fuzzy relation R is the relation among elements of A_i and described by a membership function $\Phi_R(a_1, a_2, \ldots, a_n)$. The membership function of two sets can be shown with a table called fuzzy table, or a matrix called fuzzy matrix.

3.3. Fuzzy equivalence relation

There is a kind of classical relation called equivalence relation which is often used to classify objects. It partitions a set so that each element belongs to only one cell of the partition. Accordingly, there is a fuzzy relation called fuzzy equivalence relation.

Definition 1. Assume a fuzzy relation R on $A \times A$. For $\forall a, b, c \in A$, if it is:

(1) reflexive: $\Phi_R(a, a) = 1$;
(2) symmetric: $\Phi_R(a, b) = \Phi_R(b, a)$;
(3) transitive: for $\forall \lambda \in [0, 1]$, if $\Phi_R(a, b) \geqslant \lambda$ and $\Phi_R(b, c) \geqslant \lambda$, then $\Phi_R(a, c) \geqslant \lambda$;

then R is a fuzzy equivalence relation.

The transitive property is also called T-transitivity considering its definition in t-norm (Moser, 2006), which means the class relation between objects is transitive. This property conforms to human perception of an object class. Therefore, fuzzy equivalence relation can be used to classify objects. However, in the real world often a *fuzzy similarity relation* (also called *fuzzy compatibility relation*) exists, which does not satisfy this transitivity and is generally weakly transitive (González and Marn, 1997).

In the next section (Section 4) on the proposed FER, we will see that the shortcoming of the normal fuzzy similarity relation is circumvented by the transitive closure which is closely related to the α-cut based fuzzy classification.

3.4. α-Cut based fuzzy classification

Definition 2. An α-cut of a fuzzy set is the classical set of all the elements with membership degrees greater than or equal to a threshold $\alpha \in [0, 1]$.

According to this definition, the element of an $m \times n$ α-cut matrix $\boldsymbol{R}_\alpha$, r_{ij}^α $(1 \leqslant i \leqslant m,\ 1 \leqslant j \leqslant n)$, is computed as

$$r_{ij}^\alpha = \begin{cases} 1 & \text{if } r_{ij} \geqslant \alpha, \\ 0 & \text{otherwise}. \end{cases} \tag{1}$$

Obviously, $\boldsymbol{R}_\alpha$ is a boolean matrix. Increasing α will change r_{ij}^α and lead to a coarse-to-fine classification. Therefore $\boldsymbol{R}_\alpha$ can be utilized to classify objects.

However the normal fuzzy similarity relation lacks transitivity and consequently the α-cut matrix can not be used for classification directly. Nevertheless, the matrix method for computing a fuzzy equivalence relation called transitive closure can be utilized. Detailed discussions of the matrix method and the proposed method FER are in the following section.

4. FER – fuzzy equivalence relation based fusion

In this section, the principle of the matrix method for obtaining the fuzzy equivalence relation is discussed first. Then the fast square method used in FER, the construction of the fuzzy similarity matrix as well as the procedure of FER are presented.

4.1. Principle of the matrix method

Before proceeding to the theorem which the matrix method is based on, we first introduce an important property of fuzzy similarity relation.

Lemma 1 Xu et al., 2007. *If R is a fuzzy similarity matrix, $\boldsymbol{R}^k$ is also a fuzzy similarity matrix for $\forall k \in \boldsymbol{N}$ (note: N represents the natural numbers).*

The concept of transitive closure need also be introduced in advance for the theorem which the matrix method is based on.

Definition 3. Assuming a fuzzy matrix R, the minimal transitive matrix containing R, $t(R)$, is called the transitive closure of R.

Now comes the theorem for the matrix method.

Theorem 1. *For the fuzzy similarity matrix R, its transitive closure t(R) is the minimal fuzzy equivalence matrix containing R.*

Proof. The theorem can be proved progressively in three steps based on the lemmas in (Xu et al., 2007) and the discussion in (De Meyer et al., 2004): (1) $t(\boldsymbol{R}) = \cup_{k=1}^n \boldsymbol{R}^k$; (2) $\exists k' \leqslant n : t(\boldsymbol{R}) = \boldsymbol{R}^{k'}$; and (3) $\forall l > k' : \boldsymbol{R}^l = \boldsymbol{R}^{k'}$.

(1) $t(\boldsymbol{R}) = \cup_{k=1}^n \boldsymbol{R}^k$
Let $\boldsymbol{R}^k = (r_{ij}^{(k)})_{n\times n},\ k = 1, 2, \ldots, n, n+1$. Apparently

$$r_{ij}^1 = r_{ij} \quad (i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, n).$$

Then

$$\begin{aligned} r_{ij}^{(n+1)} &= \mathop{\vee}_{m_1=1}^{n} (r_{im_1}^{(1)} \wedge r_{m_1 j}^{(n)}) = \mathop{\vee}_{m_1=1}^{n} \{r_{im_1} \wedge [\mathop{\vee}_{m_2=1}^{n} (r_{m_1 m_2} \wedge r_{m_2 j}^{(n-1)})]\} \\ &= \mathop{\vee}_{m_1=1}^{n} \mathop{\vee}_{m_2=1}^{n} (r_{im_1} \wedge r_{m_1 m_2} \wedge r_{m_2 j}^{(n-1)}) = \cdots \\ &= \mathop{\vee}_{m_1=1}^{n} \mathop{\vee}_{m_2=1}^{n} \cdots \mathop{\vee}_{m_n=1}^{n} (r_{im_1} \wedge r_{m_1 m_2} \wedge \cdots r_{m_n j}) \\ &= r_{iq_1} \wedge r_{q_1 q_2} \wedge \cdots r_{q_n j}, \end{aligned}$$

where $\{q_1, q_2, \ldots, q_n\} \subseteq \{1, 2, \ldots, n\}$. Hence, two elements of $\{q_1, q_2, \ldots, q_n\}$ must be equal.
If $q_h = q_g (1 \leqslant h \leqslant g \leqslant n)$, then

$$r_{ij}^{(n+1)} \leqslant r_{iq_1} \wedge \cdots \wedge r_{q_{h-1} q_h} \wedge r_{q_g q_{g+1}} \wedge \cdots \wedge r_{q_n j} \leqslant r_{ij}^{(p)}, \tag{2}$$

where $p = n + 1 - (g - h) \leqslant n$. If $i = q_h (1 \leqslant h \leqslant n)$, then

$$r_{ij}^{(n+1)} \leqslant r_{q_h q_{h+1}} \wedge r_{q_{h+1} q_{h+2}} \wedge \cdots \wedge r_{q_n j} \leqslant r_{ij}^{(p)}, \tag{3}$$

where $p = n + 1 - h \leqslant n$.
Eqs. (2) and (3) prove that there exists $p (1 \leqslant p \leqslant n)$ so that $r_{ij}^{(n+1)} \leqslant r_{ij}^{(p)}$ and, therefore, $r_{ij}^{(n+1)} \leqslant \mathop{\vee}_{k=1}^{n} r_{ij}^{(k)}$. This conclusion yields $\boldsymbol{R}^{n+1} \subseteq \cup_{k=1}^n \boldsymbol{R}^k$ and, furthermore, $\boldsymbol{R}^q \subseteq \bigcup_{k=1}^{n} \boldsymbol{R}^k$ $(\forall q \geqslant n + 1)$, so

$$t(\boldsymbol{R}) = \bigcup_{k=1}^{n} \boldsymbol{R}^k. \tag{4}$$

(2) $\exists k' \leqslant n : t(\boldsymbol{R}) = \boldsymbol{R}^{k'}$
For R^2,

$$r_{ij}^{(2)} = \mathop{\vee}_{j_1=1}^{n} (r_{ij_1} \wedge r_{j_1 i}) \geqslant r_{ii} \wedge r_{ij} = r_{ij}$$

given $r_{ii} = 1$ (note: R is a fuzzy similarity matrix), so that $\boldsymbol{R} \subseteq \boldsymbol{R}^2$. Consequently, we get

$$\boldsymbol{R}^3 = \boldsymbol{R}^2 \circ \boldsymbol{R} \supseteq \boldsymbol{R} \circ \boldsymbol{R} = \boldsymbol{R}^2 \tag{5}$$

and

$$\boldsymbol{R}^p = \boldsymbol{R}^{p-2} \circ \boldsymbol{R}^2 \supseteq \boldsymbol{R}^{p-2} \circ \boldsymbol{R} = \boldsymbol{R}^{p-1}, \tag{6}$$

where $\circ$ denotes the composition operation.
Eqs. (5) and (6) imply a non-decreasing matrix sequence $\boldsymbol{R}^p (p \in \boldsymbol{N})$,

$$\boldsymbol{R} \subseteq \boldsymbol{R}^2 \subseteq \boldsymbol{R}^3 \subseteq \cdots \subseteq \boldsymbol{R}^{p-1} \subseteq \boldsymbol{R}^p \subseteq \cdots \tag{7}$$

Combining Eq. (4) just proved gives

$$t(\boldsymbol{R}) = \bigcup_{m=1}^{n} \boldsymbol{R}^m = \boldsymbol{R}^n.$$

Since $\boldsymbol{R}^p$ is a non-decreasing series (Eq. (7)), there exists $k'(k' \leqslant n)$ such that $\boldsymbol{R}^{k'} = \boldsymbol{R}^{k'+1} = \boldsymbol{R}^{k'+2} = \cdots = \boldsymbol{R}^n$, i.e.,

$$t(\boldsymbol{R}) = \boldsymbol{R}^{k'}. \tag{8}$$

(3) $\forall l > k' : \boldsymbol{R}^l = \boldsymbol{R}^{k'}$

According to Eqs. (7) and (8), $\forall l > k'$,

$$t(\boldsymbol{R}) = \boldsymbol{R}^{k'} \subseteq \boldsymbol{R}^l \subseteq \bigcup_{m=1}^{\infty} \boldsymbol{R}^m = t(\boldsymbol{R}).$$

Then,

$$\boldsymbol{R}^l = \boldsymbol{R}^{k'}. \tag{9}$$

$\boldsymbol{R}^{k'}$ is a fuzzy similarity matrix (see Lemma 1) and Eq. (9) shows $t(\boldsymbol{R}) = \boldsymbol{R}^{k'}$ is transitive. Therefore, $\boldsymbol{R}^{k'}$ is the minimum fuzzy equivalence matrix containing $\boldsymbol{R}$, $t(\boldsymbol{R})$. $\square$

The proof of Theorem 1 shows that the transitive closure $t(\boldsymbol{R})$ can be compuated as a sufficiently high power of the fuzzy similarity matrix $\boldsymbol{R}$, $\boldsymbol{R}^{k'}$. $t(\boldsymbol{R})$ is then the fuzzy equivalence matrix $\boldsymbol{R}^e$ for the α-cut based fuzzy classification. This method of computing the fuzzy equivalence matrix is called matrix method. In the FER, we adopt the fast square method as a quick solution of $\boldsymbol{R}^e$.

4.2. The fast square method

The practical method to compute the fuzzy equivalence matrix based on Theorem 1 is the fast square method:

$$\boldsymbol{R} \to \boldsymbol{R}^2 \to \boldsymbol{R}^4 \to \cdots \to \boldsymbol{R}^{2^p} \to \cdots$$

After limited iterations, there exists $q \in \boldsymbol{N}$,

$$\boldsymbol{R}^{2^q} = \boldsymbol{R}^{2^{q+1}}.$$

Then the fuzzy equivalence matrix $\boldsymbol{R}^e$ which is also the transitive closure $t(\boldsymbol{R})$ can be computed as

$$\boldsymbol{R}^e = t(\boldsymbol{R}) = \boldsymbol{R}^{2^q} = \boldsymbol{R}^{2^{q+1}} \tag{10}$$

for further classification.

Example 1. Suppose a five-object set $A = \{a_1, a_2, a_3, a_4, a_5\}$. The similarities between the objects can be described by the fuzzy relation matrix

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0.8 & 0 & 0.2 & 0.1 \\ 0.8 & 1 & 0.6 & 0.2 & 0.9 \\ 0 & 0.6 & 1 & 0.5 & 0.7 \\ 0.2 & 0.2 & 0.5 & 1 & 0.5 \\ 0.1 & 0.9 & 0.7 & 0.5 & 1 \end{bmatrix}.$$

This fuzzy relation is reflexive and symmetric because $\boldsymbol{R}_{ii} = 1$ and $\boldsymbol{R}_{ij} = \boldsymbol{R}_{ji}$. However $\boldsymbol{R}^2 = \boldsymbol{R} \circ \boldsymbol{R} \neq \boldsymbol{R}$. Hence, the fuzzy relation is not transitive and it is a fuzzy similarity relation.

Without transitivity, confused classification result can be easily obtained. For example, if $\alpha = 0.8$, the α-cut matrix is

$$\boldsymbol{R}_{0.8} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

We can see that a_1 and a_2 are in the same class, and a_2 and a_5 are in the same class. But a_1 and a_5 are not in the same class, which conflicts with our notion of object class.

But the fast square method can be applied to obtain the fuzzy equivalence matrix.

$$\boldsymbol{R}^2 = \boldsymbol{R} \circ \boldsymbol{R} = \begin{bmatrix} 1 & 0.8 & 0.6 & 0.2 & 0.8 \\ 0.8 & 1 & 0.7 & 0.5 & 0.9 \\ 0.6 & 0.7 & 1 & 0.5 & 0.7 \\ 0.2 & 0.5 & 0.5 & 1 & 0.5 \\ 0.8 & 0.9 & 0.7 & 0.5 & 1 \end{bmatrix} \neq \boldsymbol{R},$$

$$\boldsymbol{R}^4 = \boldsymbol{R}^2 \circ \boldsymbol{R}^2 = \begin{bmatrix} 1 & 0.8 & 0.7 & 0.5 & 0.8 \\ 0.8 & 1 & 0.7 & 0.5 & 0.9 \\ 0.7 & 0.7 & 1 & 0.5 & 0.7 \\ 0.5 & 0.5 & 0.5 & 1 & 0.5 \\ 0.8 & 0.9 & 0.7 & 0.5 & 1 \end{bmatrix} \neq \boldsymbol{R}^2,$$

$$\boldsymbol{R}^8 = \boldsymbol{R}^4 \circ \boldsymbol{R}^4 = \begin{bmatrix} 1 & 0.8 & 0.7 & 0.5 & 0.8 \\ 0.8 & 1 & 0.7 & 0.5 & 0.9 \\ 0.7 & 0.7 & 1 & 0.5 & 0.7 \\ 0.5 & 0.5 & 0.5 & 1 & 0.5 \\ 0.8 & 0.9 & 0.7 & 0.5 & 1 \end{bmatrix} = \boldsymbol{R}^4.$$

According to Eq. (10), the fuzzy equivalence matrix is calculated as $\boldsymbol{R}^e = \boldsymbol{R}^4 = \boldsymbol{R}^8$.

Now the classification can be achieved with $\boldsymbol{R}^e$. For example, if $\alpha = 0.8$, the α-cut matrix is

$$\boldsymbol{R}^e_{0.8} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Consequently, we obtain a consistent classification result

$$A/R^e_{0.8} = \{\{a1, a2, a5\}, \{a3\}, \{a4\}\}.$$

The classification using α-cut requires the fuzzy matrix of the normal fuzzy similarity relation. Therefore, our question becomes how to compute the fuzzy matrix. We will show the method in the coming subsection.

4.3. Computing the fuzzy matrix of the fuzzy similarity relation

The candidate windows containing the same partial or full human are normally very near, therefore, the distance between the centers of neighboring windows can be used for fusion. The windows closer to each other will more likely belong to the same class than the windows further apart. Thus, the fuzzy matrix is computed based on the distances between the window centers.

Let window i be W_i and its center be (w_{xi}, w_{yi}). The coefficient r^W_{ij} for W_i and W_j can be formulated as

$$r^W_{ij} = \begin{cases} 1 & \text{if } i = j, \\ 1 - \sqrt{\frac{(w_{xi}-w_{xj})^2+(w_{yi}-w_{yj})^2}{c}} & \text{if } i \neq j, \end{cases} \tag{11}$$

where c is a positive parameter to tune the distance between W_i and W_j. If r^W_{ij} is less than zero, it is set to be zero.

4.4. The procedure of FER

Based on above discussions, FER consists of the following four steps.

Step 1: compute the fuzzy similarity relation.
Assume each candidate window W_i in the candidate window set $W = \{W_1, W_2, \ldots, W_n\}$ has one feature: its center (w_{xi}, w_{yi}). The fuzzy similarity relation R is represented by the fuzzy similarity matrix $\boldsymbol{R}_W$ whose element r_{ij}^W representing the similarity between W_i and W_j is computed by (11).

Step 2: construct the fuzzy equivalence relation.
The fuzzy equivalence matrix $\boldsymbol{R}^e$ can be computed from $\boldsymbol{R}_W$ with the matrix method based on Theorem 1. The fast square method discussed in Section 4.2 is used for the calculation.

Step 3: classify the windows progressively.
The fuzzy classification of candidate windows is based on the α-cutted equivalence matrix $\boldsymbol{R}_\alpha^e$ computed from $\boldsymbol{R}^e$ by (1). Windows W_i and W_j will be assigned to the same class if r_{ij}^α in $\boldsymbol{R}_\alpha^e$ equals to 1.

Step 4: compute the final detection window.
The detection window is finally obtained by the weighted average of the top-left coordinates and sizes of all clustered candidate windows. The weights are assigned according to the detection score of each candidate window.

5. Experimental results

We now discuss the experimental results using FER. NMS and BR are included to compare the performances between our method and previous methods. The experimental images are from the MIT pedestrian dataset (MIT Center for Biological, Computational Learning, and MIT, 2000) and INRIA person dataset (Dalal and Triggs, 2005). The simple cell-structured LBP (Wang et al., 2009) features are used as the feature descriptor. The open LIBSVM (Chang and Lin, 2011) is used as the SVM classifier to train 500 positive and 1000 negative samples from each dataset. For the sliding window, its horizontal and vertical steps are set to be eight pixels.

Two types of comparison are carried out for the three methods. One is to test the same image with different numbers of candidate windows (Type 1). The other type is to test different images with the same number of candidate windows (Type 2). According to the principle of fuzzy matrix construction, the size of fuzzy matrix equals to the number of candidate windows involved. The fusion result and runtime of each method are presented. The count of runtime for each method begins when all candidate windows' centers are input into the method and stops when the clustered window centers are obtained, e.g., the time consumed from Step 2 to 4 for FER.

5.1. Type 1: performance comparison of the same image with different candidate windows

Fig. 1 shows the MIT dataset trained Type 1 fusion comparison where the number of candidate windows are from 4 to 20. No method is significantly better than the others, i.e., FER obtains almost the same detection accuracy as NMS.

However their fusion speeds are considerably different as shown in Table 1. NMS is slowest and its speed increases with more and more candidate windows. BR and FER are much better than NMS. Unlike NMS, their speeds do not increase significantly when the number of candidate windows increases. FER is consistently faster than BR. In addition, The average speed of FER is 0.0584 s, which is almost half of that of BR (0.1170 s). Therefore, FER is fastest among the three methods.

Fig. 2 shows the INRIA dataset trained Type 1 fusion comparison. The test image and the number of candidate windows are the same as those of Fig. 1. The backgrounds in the INRIA dataset are generally more complex than these in the MIT dataset, which affect the accuracy of the human detector. Therefore, some tests (#candidate window being 8, 10, 12, 14 and 16) input with more

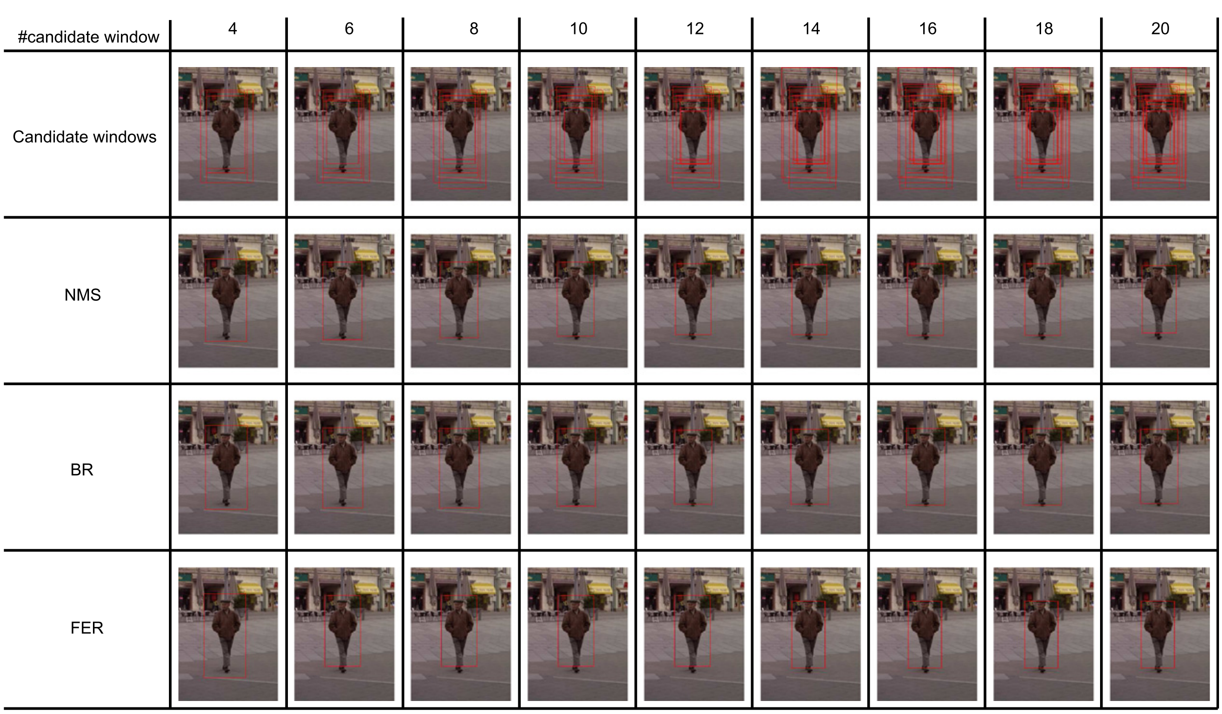

Fig. 1. The Type 1 fusion comparison trained with MIT dataset. The first row gives the number of candidate windows input. The corresponding candidate windows are shown in the second row. The third to fifth row show the fusion result of NMS, BR and FER, respectively.

Table 1
The runtimes (in seconds) for the experiment shown in Fig. 1.

#Candidate window	4	6	8	10	12	14	16	18	20
NMS	13.4493	35.1047	77.5987	125.6900	204.2647	317.1164	450.5593	535.6379	719.4465
BR	0.1145	0.1137	0.1146	0.1182	0.1186	0.1176	0.1167	0.1220	0.1168
FER	0.0577	0.0592	0.0589	0.0559	0.0577	0.0594	0.0585	0.0592	0.0592

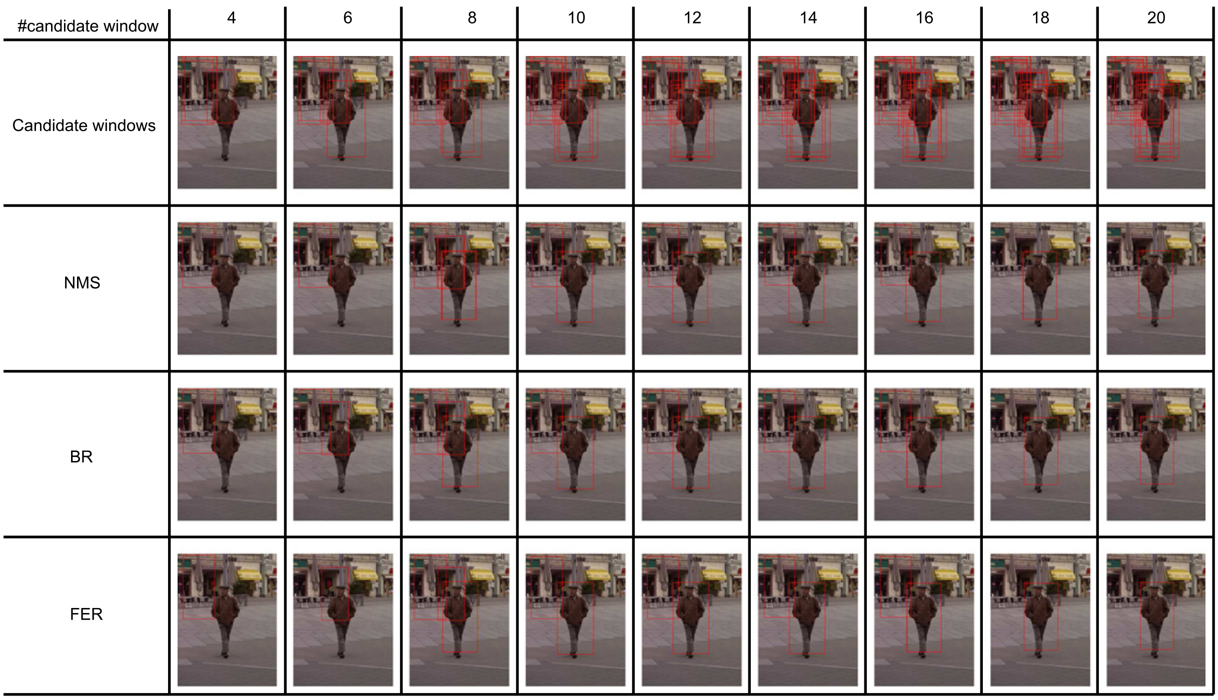

Fig. 2. The Type 1 fusion comparison trained with INRIA dataset. The test image is the same as shown in Fig. 1. The purpose of each row as well as the meaning of each label are the same as those of Fig. 1.

Table 2
The runtimes (in seconds) for the experiment shown in Fig. 2.

#Candidate window	4	6	8	10	12	14	16	18	20
NMS	6.1141	13.6151	34.7494	69.5113	97.8865	145.0071	196.0140	251.4897	300.9471
BR	0.1050	0.1078	0.1065	0.1100	0.1054	0.1080	0.1077	0.1083	0.1091
FER	0.0539	0.0533	0.0530	0.0530	0.0540	0.0532	0.0537	0.0549	0.0541

Fig. 3. The Type 2 fusion comparison trained with MIT dataset. There are ten test images, each with eight candidate windows. The fusion results of NMS, BR and FER are shown in the second to fourth rows respectively.

Table 3
The runtimes (in seconds) of NMS, BR and FER for the experiment shown in Fig. 3.

Image no.	1	2	3	4	5	6	7	8	9	10
NMS	47.9216	52.8507	64.4433	77.5987	61.9300	61.54134	41.9526	55.7686	57.6088	30.1305
BR	0.1159	0.1165	0.1180	0.1146	0.1191	0.1201	0.1184	0.1150	0.0927	0.0988
FER	0.0589	0.0599	0.0590	0.0589	0.0573	0.0577	0.0574	0.0581	0.0482	0.0552

Table 4
The runtimes (in seconds) of NMS, BR and FER from the Type 2 fusion comparison trained with the INRIA dataset. The same ten images with the same number (eight) candidate windows as Fig. 3 are used.

Image no.	1	2	3	4	5	6	7	8	9	10
NMS	39.4643774	54.9320	49.5057	34.7494	141.0227	52.2745	72.7330	42.5467	23.2706	26.5503
BR	0.1064	0.1090	0.1056	0.1065	0.1057	0.1064	0.1051	0.1057	0.0891	0.0853
FER	0.0577	0.0535	0.0531	0.0530	0.0531	0.0536	0.0542	0.0536	0.0427	0.0434

than one candidate window clusters (check the candidate windows row of Fig. 2) detected by the human detector. For them, each fusion method can easily obtain two or more fused windows. Again, we can conclude that FER is almost as accurate as NMS and BR.

The runtimes of the comparison of Fig. 2 are listed in Table 2. Similar to those in Table 1, NMS is significantly slow and FER is always the fastest among the three. The runtimes of FER and BR vary little, and FER (average runtime being 0.0537 s) is also almost half of BR (average runtime being 0.1075 s) in speed.

5.2. Type 2: performance comparison with different images having the same candidate windows

Fig. 3 show the Type 2 comparison trained with MIT dataset. There are ten test images, each with eight candidate windows. Like the previous experimental comparisons, this figure indicates no apparent performance difference among NMS, BR and FER.

For runtimes (Table 3), their speed rank keeps the same as previous experiments, where FER and NMS are the fastest and the slowest respectively. FER (0.0571 s) is still about half of the BR (0.1129 s) in average speed. In addition, the speed of NMS changes significantly from 30.1305 s (no. 10) to 77.5987 s (no. 4), while the other two methods' vary little.

The same ten images with the same number of candidate windows as those of Fig. 3 are also tested with INRIA dataset. Again, there is no significant accuracy difference among NMS, BR and FER. Table 4 shows the runtimes of each method in the experiment. The same rank summary about NMS, BR and FER as the previous fusion comparisons can be obtained.

6. Conclusions

This paper proposes a novel and fast window fusion method called FER for human detection based on fuzzy equivalence relation. We present its working principle from related fuzzy set theories and formulate the FER algorithm. Experimental results show that the proposed method runs much faster than established methods like NMS and BR, while achieving comparable detection accuracy.

Future work includes studying an efficient threshold learning scheme for α-cut. Current α-cut is implemented with the empirical threshold value. An automatic estimation method might be better. Efficient output of candidate windows from a human detector is also one of our future directions. Our experiments (e.g., Fig. 2) indicate the accuracy of the fusion method depends on the accuracy of the candidate windows. We may gain wisdoms of solving it from the tons of researches on this open problem.

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